Q-balls in a $U(1)$ gauge theory coupled to $U(1) \times U(1)$ symmetric scalars

Based on 2008.09844 [Phys. Rev. D 102, 076017 (2020)] and 2011.01634

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Outline

Why Q-balls?

Theoretical motivation: solitons in 3d

Derrick's theorem

- consider scalar fields with "usual" action
- rescaling $\phi_\lambda(x) = \phi(\lambda x)$: scaling of energy terms
- $\partial E/\partial\lambda = 0$
- no finite-energy, purely scalar solitons in $d > 2$

Hobart 1963, Derrick 1964, Rosen 1966

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Evade DT?

- Infinite energy (cosmic strings)
- Higher spin (e.g., gauge) fields (monopoles)
- Higher derivatives (Skyrmions)
- Time-dependent fields (Q-balls)

Kibble 1976, 't Hooft 1974, Polyakov, 1974, Skyrme 1961, Rosen 1968, Coleman 1985

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Important consequence: stability

- particle number

$$N = Q/q$$

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- bound if

$$E < E_{\text{free}}\,, \quad E_{\text{free}} = mN$$

Rosen 1968, Coleman 1985, Lee & Pang 1992

Physics of Q-balls

- Q-balls in SM extensions **Kusenko 1997**
- Q-balls as Dark Matter **Frieman, Gelmini, Gleiser & Kolb 1988; Kusenko & Shaposhnikov 1998**
- Role in baryogenesis **Dodelson & Widrow 1990, Enqvist & McDonald 1998**

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- Screening in the Abelian Higgs model
- Interior of screened Q-balls homogeneous
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Ishihara & Ogawa, 2018, 2019

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Self-interaction?

Limiting cases?

The model

$$S = \int \mathrm{d}^4 x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi^* D^\mu \phi + D_\mu \psi^* D^\mu \psi - V \right]$$

- ϕ Higgs, complex scalar, $\langle \phi \rangle \neq 0$
- ψ matter, complex scalar, $\langle \psi \rangle = 0$
- A_μ gauge field

$g = \mathrm{diag}(+,-,-,-)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu$, $D_\mu \phi = (\partial_\mu - \mathrm{i} e_1 A_\mu)\phi$, $D_\mu \psi = (\partial_\mu - \mathrm{i} e_2 A_\mu)\psi$

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Potential: most general $U(1) \times U(1)$ with $\langle \phi \rangle \neq 0$, $\langle \psi \rangle = 0$:

$$V = \frac{\lambda_1}{2}(|\phi|^2 - \eta^2)^2 + \frac{\lambda_2}{2}|\psi|^4 + \lambda_{12}(|\phi|^2 - \eta^2)|\psi|^2 + m^2|\psi^2|$$

Forgács & ÁL 2016

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Rescaling:

$\eta \to 1$, $e_i \to q_i = e_i/e$, $\lambda_{1,2,12} \to \beta_{1,2,12} = \lambda_{1,2,12}/e^2$, $\mu = m^2/(e^2\eta^2)$

Ansatz

Spherically symmetric solution

$$\boxed{A_0 = \alpha(r)\,, \quad \phi = f_1(r)\,, \quad \psi = \mathrm{e}^{\mathrm{i}\omega t} f_2(r)}$$

α, $f_{1,2}$ *profile functions*, solved for numerically

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α, $f_{1,2}$ *profile functions*, solved for numerically

- radial equations from Action S
- boundary conditions at $r = 0$ from regularity

$$f_{1,2} \sim f_{1,2}(0) + f_{1,2}^{(2)} r^2 + \dots\,, \qquad \alpha \sim \alpha(0) + \alpha^{(2)} r^2 + \dots$$

- boundary conditions at $r \to \infty$: approach vacuum

$$f_1 \to 1\,, \qquad f_2 \to 0\,, \qquad \alpha \to 0$$

Energy and charges

Energy of spherical configuration

$$E = \frac{4\pi}{e}\eta \int_0^\infty \mathrm{d}r r^2 \left[(f_1')^2 + (f_2')^2 + \frac{1}{2}(\alpha')^2 + q_1^2\alpha^2 f_1^2 + (q_2\alpha - \omega)^2 f_2^2 + V \right]$$

where

$$V = \frac{\beta_1}{2}(f_1^2 - 1)^2 + \frac{\beta_2}{2} f_2^4 + \beta_{12}(f_1^2 - 1) f_2^2 + \mu f_2^2$$

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Charges: $Q_{\phi,\psi} = \int 4\pi r^2 \mathrm{d}r \rho_{\phi,\psi}$

$$\rho_\phi = 2 q_1^2 \alpha f_1^2 \,, \qquad \rho_\psi = 2 q_2 (q_2\alpha - \omega) f_2^2 \,.$$

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Both conserved. Perfect charge screening (Gauss' thm):

$$Q_\phi + Q_\psi = 0$$

$\rightarrow$ test of numerical solution

Forgács & ÁL 2020

Effective action

$$S_{\rm eff} = I_1 - I_3\,, \quad I_1 = 4\pi\int \mathrm{d}r r^2 K_{\rm eff}\,, \quad I_3 = 4\pi\int \mathrm{d}r r^2 U_{\rm eff}$$

kinetic term:

$$K_{\rm eff} = (f_1')^2 + (f_2')^2 - (\alpha')^2/2\,,$$

effective potential

$$\boxed{\begin{aligned} U_{\rm eff} = &-\beta_1(f_1^2-1)^2/2 - \beta_2 f_2^4/2 - \beta_{12}(f_1^2-1)f_2^2 - \mu f_2^2 \\ &+ q_1^2\alpha^2 f_1^2 + (q_2\alpha-\omega)^2 f_2^2 \end{aligned}}$$

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Virial argument $(r \to \lambda r)$: $I_1 = 3 I_3$,

$$\frac{E}{\eta} = -\omega \frac{Q_\psi}{q_2} + \frac{2}{3e} I_1$$

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Asymmetry in ϕ, ψ: gauge choice ($Q_\phi = -Q_\psi$)

Forgács & ÁL 2020

Domain of existence

For other parameters fixed:

$$\omega_{\min} < \omega < \omega_{\max}$$

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$\omega_{\min}$:

- Interior of solution: "true" vacuum of U_{eff}
- Exterior of solution: "false" vacuum of U_{eff} (true vac.)
- at $\omega = \omega_{\min}$ U_{eff}("true vac") $=$ U_{eff}("false" vac)

Domain of existence

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- Exterior of solution: "false" vacuum of $U_{\rm eff}$ (true vac.)
- at $\omega = \omega_{\rm min}$ $U_{\rm eff}$("true vac") $= U_{\rm eff}$("false" vac)

$\omega_{\rm max}$

- asymptotic solution $f_2 \sim \exp(-\sqrt{\mu - \omega^2} r)/r$

$$\omega_{\rm max}^2 = \mu$$

+ positivity conditions, $\beta_1 < \beta_{12}/2$ ($q_1 = q_2$)

Radial equations

Ansatz, $\delta S_{\text{eff}} = 0$:

$$\frac{1}{r^2}(r^2 f_1')' = f_1\left[-q_1^2\alpha^2 + \beta_1(f_1^2 - 1) + \beta_{12} f_2^2\right]$$

$$\frac{1}{r^2}(r^2 f_2')' = f_2\left[-(q_2\alpha - \omega)^2 + \beta_2 f_2^2 + \mu + \beta_{12}(f_1^2 - 1)\right]$$

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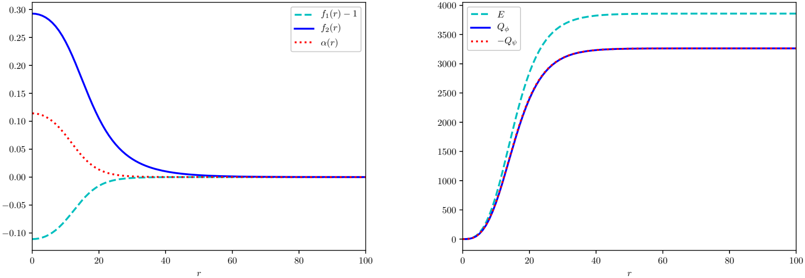
Boundary conditions

- $f_{1,2}(0) = \alpha(0) = 0$
- $f_1(\infty) = 1,\ f_2(\infty) = \alpha(\infty) = 0$

Numerical solution:

- large interval $0 \ldots L$
- collocation, `COLNEW` package (Ascher 1987)

Numerical solution



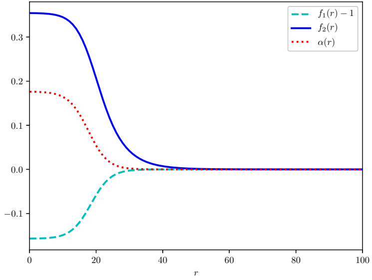
$$\beta_1 = 0.5,\ \beta_{12} = \mu = 1.4,\ \beta_2 = 0.25,\ \omega = 1.180$$

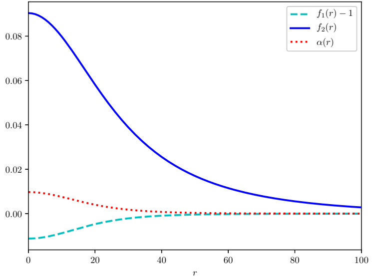
- $\beta_2 \neq 0$ does not change much
- charge cancellation local

Method: collocation, error estimate: 2×10^{-6}

Varying ω

$$\beta_1 = 0.5,\ \beta_{12} = \mu = 1.4,\ \beta_2 = 0$$





$\omega = 1.174$
Approaching ω_{min}
Whole Q-ball core expands

$\omega = 1.183$
Approaching ω_{max}
ψ component "tail" becomes long

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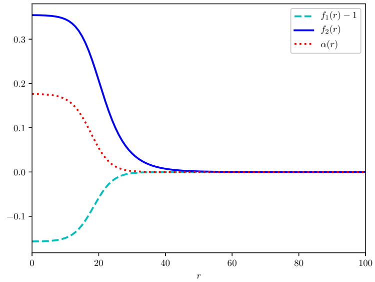

$\omega = 1.174$
Approaching $\omega_{\min}$
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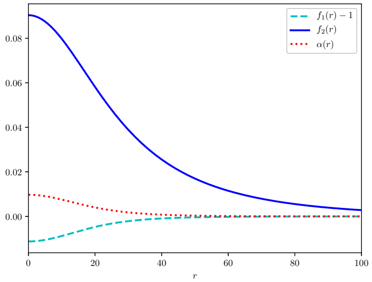

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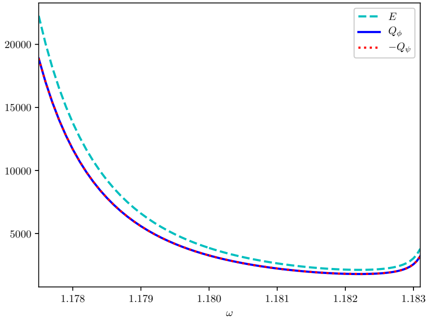
Changing other parameters: $\omega_{\min}$ or $\omega_{\max}$

E & Q vs. ω

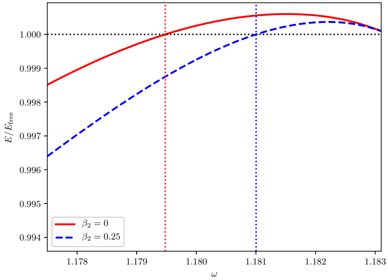


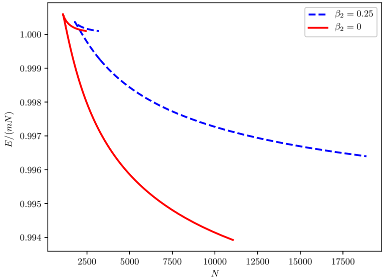
$\beta_1 = 0.5$, $\beta_{12} = \mu = 1.4$, and $\beta_2 = 0.25$

Energy and charge diverges at both limits

Very similar for $\beta_2 = 0$ and $\beta_2 \neq 0$

Stability: $E/E_{\rm free}$





$$\beta_1 = 0.5,\ \beta_2 = 0.25 \text{ and } 0,\ \beta_{12} = \mu = 1.4$$

$$N = Q_\psi / q_2\,, \qquad E_{\rm free} = mN = \sqrt{\mu} N$$

Stable branch for large N, Q (other branch not energetically favourable)

Ishihara & Ogawa, 2019

$q_1 \neq q_2$, limiting cases

Small q_1

- Positivity condition $\beta_1 < \mu q_1^2/2$
- $q_1 = 0$ cannot be reached
- distinct family of solutions ($q_1 = 0$ Lee & Yoon 1989)

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Small q_2

- a quite simple limit
- in the limiting case, $\alpha \to 0$
- reproduces known result (Friedberg, Lee & Sirlin, 1979)

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$\beta_{1,2} \to 0$

Cusp on $E/E_{\rm free}$ vs. N not observed

Summary

- Q-balls: nontopological solitons with time-periodic scalars
- Screened, gauged Q-balls extended to most general $U(1) \times U(1)$ symmetric scalar potential
- limiting cases $q_1 \to 0$, $q_2 \to 0$, $\beta_{1,2} \to 0$
- depending on parameters: 2 distinct families of Q-balls

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- depending on parameters: 2 distinct families of Q-balls

THANK YOU FOR YOUR ATTENTION!

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Screening in the Abelian Higgs model

Abelian Higgs model (A,ϕ) & external charge$\rho_{\rm ext}$
Global screening: consequence of Gauss' theorem:

$$\int \mathrm{d}^3x(m_A^2 A^0 - \rho_{\rm ext} - \rho_\phi) = -\int \mathrm{d}^3x \nabla^2 A^0 = \int \mathrm{d}^2x \partial_n A^0 = 0$$

Perturbation theory: $\phi = \eta + \chi/\sqrt{2}$,

$$A_0^{(1)} = \epsilon A_0^{(1)} + \epsilon^2 A_0^{(2)} + \dots, \quad \chi = \epsilon^2 \chi^{(2)} + \dots$$

$$(\nabla^2 - m_s^2)\chi^{(k)} = -\xi^{(k)}, \quad (\nabla^2 - m_A^2) A_0^{(k)} = -\sigma_0^{(k)}$$

with

$$\xi^{(1)} = 0, \qquad \sigma_0^{(1)} = \rho_{\rm ext}^{(1)},$$

$$\xi^{(2)} = e^2 v A_\mu^{(1)} A^{(1)\mu}, \qquad \sigma_0^{(2)} = -2e^2 v \chi^{(1)} A_0^{(1)},$$

Order-by-order cancellation:

Solution using Green's functions:

$$A_0^{(k)}(x_i) = \int \mathrm{d}^3x' \, G_A(x_i - x_i')\sigma_0^{(k)}(x_i') \,, \qquad G_A(\mathbf{x}) = \frac{1}{4\pi|\mathbf{x}|} \exp(-m_A|\mathbf{x}|) \,,$$

$$\chi^{(k)}(x_i) = \int \mathrm{d}^3x' \, G_s(x_i - x_i')\xi^{(k)}(x_i') \,, \qquad G_s(\mathbf{x}) = \frac{1}{4\pi|\mathbf{x}|} \exp(-m_s|\mathbf{x}|) \,.$$

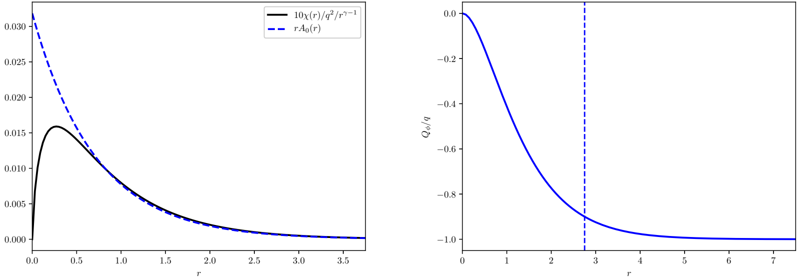
Consequently,

$$Q_A^{(k)} = -\int \mathrm{d}^3x m_A^2 A^{(k)} = -m_A^2 \int \mathrm{d}^3x \mathrm{d}^3x' \, G_A(x_i - x_i')\sigma_0^{(k)}(x_i') = -Q_\phi^{(k)}$$

Including $Q_A^{(1)} = -Q_{\mathrm{ext}}^{(1)}$

Point charge

Point charge: $\rho_{\rm ext} = q\delta^3(\mathbf{r})$



$$e = 1,\ \lambda = 2.0,\ q = 0.4$$

$$A_0^{(1)}(r) = \frac{1}{4\pi r}\mathrm{e}^{-m_A r}\,,$$

$$\chi^{(2)}(r) = -\frac{e^2 v}{2(4\pi)^2 m_s r}\Bigg[\mathrm{e}^{-m_s r}\left(\mathrm{Ei}[(m_s - 2m_A)r] - \log\frac{|m_s - 2m_A|}{m_s + 2m_A}\right) - \mathrm{e}^{m_s r}\,\mathrm{Ei}[-(m_s + 2m_A)r]\Bigg]\,.$$

Point charges

Numerical and leading order agrees within line width

Perturbative solution to calculate interaction between point charges
Two length scales: $1/m_A$ (screening) and $1/m_s$ (scalar pertrubations)

Type II: $m_s > m_A$: due to gauge field

$$V_{\mathrm{II}}(r) = \frac{q_1 q_2}{4\pi r} \mathrm{e}^{-m_A r}$$

Type I: $m_s < m_A$: due to scalar field

$$V_{\mathrm{I}}(r) = \frac{e^4 v^2 q_1^2 q_2^2}{4(4\pi)^3 m_s m_A} \log \frac{2m_A - m_s}{2m_A + m_s} \frac{\mathrm{e}^{-m_s r}}{r}$$

For type I: like charges attract!

Analogy: superconductivity; method: Speight, 1997

Forgács & ÁL, 2020