An algorithm for permutationally invariant state reconstruction for larger qubit numbers

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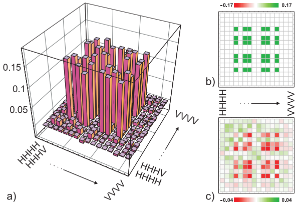
Quantum State Tomography

- **Task**: Determine a previously unknown quantum state ρ_0
  - ⋆ Measurement scheme
  - ⋆ State reconstruction part



- Measurement effort of standard tomography schemes increases exponentially with the particle number (since they work for all quantum states).
- New protocols [Compressed sensing, PI-Tomography] reduce this cost, because they are tailored to special classes of states.

Permutationally invariant tomography

- Class of states: Permutationally invariant states satisfy

$$\rho_{\mathrm{PI}} = V(p)\rho_{\mathrm{PI}}V(p)^\dagger$$

  for all possible permutations p of N particles.

- Measurement scheme for N-qubits: For each setting $\hat{s} \in \mathbb{R}^3$ one measures in the eigenbasis of $\hat{s} \cdot \vec{\sigma} = |0\rangle_s\langle 0| - |1\rangle_s\langle 1|$ and uses only the coarse-grained outcomes

$$M_{k|s} = \left[|0\rangle_s\langle 0|^{\otimes N-k} \otimes |1\rangle_s\langle 1|^{\otimes k} \right]_{\mathrm{PI}}, \forall k = 0, \ldots, N.$$

- Measurement effort:

$$\left. \begin{array}{l} (N^2+3N+2)/2 \text{ settings} \\ N+1 \text{ outcomes} \end{array} \right\} \quad \Rightarrow \quad \text{cubic scaling}$$

State reconstruction

- Real probabilities $P(k|s) = \mathrm{tr}(\rho_0 M_{k|s})$ can only be approximated by relative frequencies $f_{k|s} = n_{k|s}/N_s$ in an experiment.

⇒ Problems in actual reconstruction

$$\mathrm{tr}(\hat{\rho}_{\mathrm{lin}} M_{k|s}) = f_{k|s}$$

since the solution (if any) $\hat{\rho}_{\mathrm{lin}} \ngeq 0$ is often not a valid state.

- **Statistical state reconstruction**: Reconstructed state $\hat{\rho}$ is the unique optimum of a fit-function $F(\rho; f_{k|s})$,

$$\hat{\rho} = \arg\min_{\rho \geq 0} F(\rho; f_{k|s}).$$

- Common reconstruction functions are maximum likelihood or least-squares methods.

Permutationally invariant state reconstruction

- State reconstruction

$$\hat{\rho}_{\mathrm{PI}} = \arg\min_{\rho_{\mathrm{PI}} \geq 0} F[\rho_{\mathrm{pi}}, f_k]$$

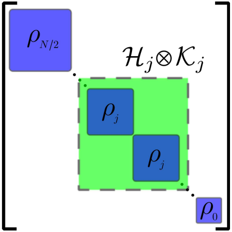
  needs to be solved for **large particle numbers**! Otherwise the PI Tomography protocol is useless for experiments.

- Two major challenges:
  - **Large dimensions**
    ⇒ Reduction method [next slide]
  - **Optimization**
    ⇒ Convex optimization
    + Systematic approach for any convex fit-function, achieves global optimum, good error control on algorithm, high accuracy and good convergence rate
    – More work

Reduction - PI toolbox

Large dimensions are handled by exploiting a particular form of permutationally invariant states/operators [Spin-coupling]:

$$\begin{aligned} \rho_{\mathrm{PI}} &= \bigoplus_{j=j_{\min}}^{N/2} p_j \rho_j \otimes \frac{\mathbb{1}}{\dim}, \\ M_{k|s} &= \bigoplus_{j=j_{\min}}^{N/2} M_{k|s;j} \otimes \mathbb{1}. \end{aligned}$$

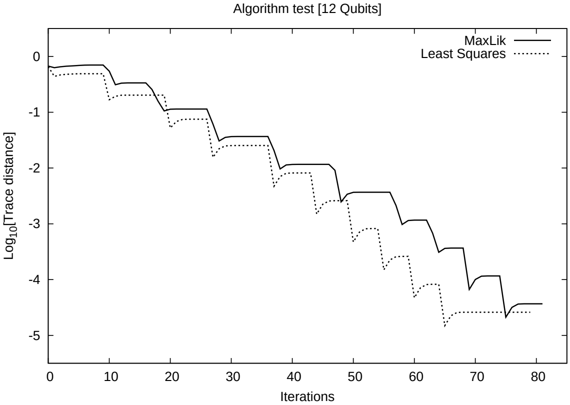
$\rho_{\mathrm{PI}} =$ 

i: Efficient storage since all ρ_i need roughly N^3 parameters.

ii: Operational way to characterize states, since ρ_{PI} is a state if and only if all ρ_j are states (and p_j valid probabilities).

iii: Efficient way to compute expectation values since $M_{k|s;j}$ can also be computed efficiently.

Current performance



	$N = 8$	$N = 10$	$N = 12$	$N = 20$
MaxLik	20 sec	60 sec	2 min	25 min
Least-Squares	15 sec	40 sec	70 sec	15 min