Supplemental Material for
"Entanglement and extreme planar spin squeezing"

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The supplemental material contains a brief description of the experiment creating planar squeezed states.

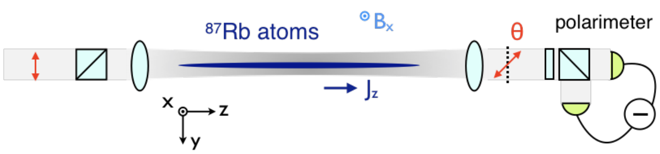

FIG. S1. The experimental configuration of Ref. [36]: an ensemble of laser-cooled ^{87}Rb atoms trapped in a singe-beam optical dipole trap and precessing in the y–z plane due to an external magnetic field B_x. The atoms are probed via paramagnetic Faraday rotation: the polarization of input linearly polarized optical pulses rotates by an angle $\theta \propto J_z$, the spin projection onto the measurement axis, as it passes through the atoms, and is detected using a balanced polarimeter.

Description of Experiment.—Experimental data is taken from Ref. [36]. In this experiment, an ensemble of laser-cooled, spin-1 Rb^{87} atoms was loaded into a single-beam optical dipole trap, polarized via optical pumping, and allowed to precess in the (y, z)-plane under an external magnetic field at a rate $\omega_{\rm L} \simeq 2\pi\times 26$ kHz. Measurement-induced spin squeezing of the $(\Delta J_y)^2$ and $(\Delta J_z)^2$ was achieved via Faraday rotation probing using a train of near-resonant, μs-duration optical pulses.

More concretely, the collective spin oscillates such that $J_z(t) = J_z \cos\phi - J_y \sin\phi$, where $\phi = \omega_{\rm L} t$. The atoms and light interact via the hamiltonian $H = gS_z J_z(t)$, where the Stokes operators S_k describe the optical polarization. This describes a quantum non-demolition (QND) measurement of instantaneous spin projection $J_z(t)$: the optical polarization rotates by an angle $\theta = gJ_z(t)$, where g is a coupling constant, proportional to the instantaneous spin projection along the z-axis [45]. Measurement of θ projects the atoms onto a state with $(\Delta J_z(t))^2$ reduced by a factor $\sim 1/(1 + g^2Nn)$, where N is the number of atoms in the ensemble, and n is the number of photons in a single probe pulse. Correspondingly, $(\Delta J_x(t))^2$ is increased by a factor $\sim 1 + g^2n$, and $(\Delta J_y(t))^2$ is increased by a negligible factor of order 1. Repeated QND measurements of $J_z(t)$ as the spins oscillate progressively squeezes the input J_z and J_y spin components, to produce the PQS state. At the same time, off-resonant scattering of probe photons during the measurement leads to decay of the spin polarization at a rate $\eta \propto g^2$, and introduces noise $\beta \propto n$ into the atomic spin components. This leads to a trade-off between measurement-induced squeezing and decoherence, and an optimum measurement strength, characterized by the total photon number $N_{\rm L} = pn$, where p is the number of probe pulses.

In the experiment, the PQS state was detected by recording a series of measurements $\theta(t_k)$ and fitting using a free induction decay model

$$\theta(t) = g\Big[J_z(t_{\rm e})\cos\phi - J_y(t_{\rm e})\sin\phi\Big]e^{-t_{\rm r}/T_2} + \theta_0, \quad \text{(S1)}$$

where $t_{\rm r} \equiv t - t_{\rm e}$ and the phase $\phi = \omega_{\rm L} t_{\rm r}$. This model allows a simultaneous estimation of $\mathbf{J} = \{J_z(t_{\rm e}), J_y(t_{\rm e})\}$ producing a conditional PQS state at time $t_{\rm e}$. $t_{\rm e}$ can be adjusted, allowing to study how the spin squeezing and entanglement evolves during the measurement as a function of $N_{\rm L}$. Conditional spin squeezing was detected by comparing two estimates, $\mathbf{J}_1$ and $\mathbf{J}_2$, taken from the set of measurements immediately before and after $t_{\rm e}$, and computing the conditional covariance matrix $\Gamma_{\mathbf{J}_2|\mathbf{J}_1} = \Gamma_{\mathbf{J}_2} - \Gamma_{\mathbf{J}_2\mathbf{J}_1}\Gamma_{\mathbf{J}_1}^{-1}\Gamma_{\mathbf{J}_1\mathbf{J}_2}$ which quantifies the error in the best linear prediction of $\mathbf{J}_2$ based on $\mathbf{J}_1$, where $\Gamma_{\mathbf{v}}$ indicates the covariance matrix for vector $\mathbf{v}$, and $\Gamma_{\mathbf{uv}}$ indicates the cross-covariance matrix for vectors $\mathbf{u}$ and $\mathbf{v}$. The measurement sequence was repeated 453 times to acquire statistics. Measurement read-out noise Γ_0 was quantified by repeating the measurement sequence without atoms in the trap. The atomic spin covariance matrix was then estimated as $\Gamma = \Gamma_{\mathbf{J}_2|\mathbf{J}_1} - \Gamma_0$, which has entries $\Gamma_{ij} = \langle J_iJ_j + J_jJ_i\rangle/2 - \langle J_i\rangle\langle J_j\rangle$.