Some novel analogic CNN algorithms for object rotation, 3D interpolation-approximation, and a “door-in-a-floor” problem

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Abstract - In this paper three interesting analogic CNN algorithms are presented for three tasks. The first task is to move a given 2D object along a prescribed path, the second task is the approximation of 3D surfaces by various interpolation and approximation methods and the third task is a specific detection problem. In this detection problem our task is to detect a “door-in-a-floor” by finding the handle and possibly the place of a text or symbol on the door. The solution methods of the tasks are summarized below.

1. Object rotation

In this part we introduce an idea how to implement some symmetrical space variant transformations such as rotation without major modification of the regular structure and casting aside the fast and simple programmability of the CNN Universal Machine [2].

Let us consider the following discrete time state equation (1-1):

\[
x_{(k+1),\text{ref}} = (1-h)x_{k,\text{ref}} + h[\sum_r \tilde{a}_{i,j,kl} \cdot v_{yj,\text{ref}} + \sum_r b_{ki} \cdot v_{ui,\text{ref}} ] + I
\]

\[
\tilde{a}_{i,j,kl} = \tilde{f}(v_{yj,\text{con}} - v_{ykl,\text{con}})
\] (1-1)

These equations describe a 2-layer CNN structure with space invariant control template, B and space variant feedback template, \( \tilde{A}_{kl} = [\tilde{a}_{i,j,kl}] \). The first layer is the control layer (con) whose input contains the circular pathway on which the object is to be moved, and the second layer’s state is the reference layer (ref) which carries the picture of the object (in this simple case the single black pixel) to be moved.

The space variance in \( \tilde{A} \) can be encoded in the following way:

The control and the reference layer are of same size and resolution and we assign a pixel on the control layer to each pixel of the reference layer. The template that is calculated on the control layer will affect the reference pixel value only in this very position. Each template value is calculated as an output of a nonlinear function (see Fig. 1 (b)). The input of this function is the difference of the actual control layer pixel and its neighbors within the given neighborhood, (in our case this is 1).

The control picture now is a circular step function with stepsize \( \delta \) (Fig. 1 (a), Fig. 2), which determines the path and direction of movement. In this case the position dependent differences are (1-2):

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\[ v_{ijcon} - v_{kcon} = \begin{cases} 0 \\ \delta \\ -\delta \end{cases} \quad (1-2) \]

These values are, according to the control image \( \delta \) and \( -\delta \) respectively on the path, 0 otherwise. The nonlinear function assigns the properly adjusted template values to these differences:

\[ \tilde{f}(v_{ijcon} - v_{kcon}) = \begin{cases} 0 \\ 1 \\ -1 \end{cases} \quad (1-3) \]

![Figure 1 (a)](image)

(a). A slice of the control image, (b), the nonlinear function that assigns the proper values to the relevant template elements.

The space variant template values for circular CCD:

\[ \bar{A}_{ref} = \begin{bmatrix} a_{ij} \end{bmatrix} \quad i = 1,2,3; \quad j = 1,2,3 \]

\[ a_{22} = 2 \]

\[ a_{ij} = \tilde{f}(v_{ijcon} - v_{kcon}) \quad i \neq 2, \quad j \neq 2 \quad (1-4) \]

![Figure 2](image)

(a) Control picture and its 3D representation for moving a black point on circular pathway.

2. Surface interpolation

In surface interpolation, we use knowledge of the image at known points. The surface is interpolated as a mathematical function.

A physical model for the potential energy function is the potential energy of the cell when the template values for the cellular neural network are to be determined. The energy function is given by:

\[ E = \iint \left( f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2 \right) \]

We prove that the matrix \( A \) is a 5x5 matrix for the gradient method. The matrix \( A \) is the following:

\[ A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -4 & 16 \\ -2 & 16 & -39 \\ 0 & -4 & 16 \\ 0 & 0 & -2 \end{bmatrix} \]

Suppose, we have a surface at grid points and we want to find the changing of the inner state of a cell to fit a fitted surface as the result.

For example, we can determine the coefficients in such a flat surface on the grid points, practically we get the following...
It is possible to introduce speed adjustment with more complicated nonlinear functions, as well as implementation of some other space variant tasks with exploitable symmetry properties on the CNN universal machine.

2. Surface interpolation and approximation

In surface interpolation problems a surface is constructed that fits exactly the set of known points. The smoothest of the fitting surfaces is chosen. So we had to construct a mathematical function, that characterize the smoothness of the surface.

A physical model was used to choose the smoothest surface. We determine the potential energy function of a thin plate fitting the set of given points. In equilibrium, the potential energy of a mechanical system is minimal, this gives the smoothest surface. A cellular neural network is used to find the minimum of this energy function. Template values are to be determined. If the surface is described by \( z = f(x,y) \), then this energy function is the following:

\[
E = \iint (f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2) \, dx \, dy
\]  

We prove that if the matrix \( B \) is zero, the current \( I \) is zero, the nonlinearity \( f(x) = x \), the matrix \( A \) is 5x5 and space variant, then the template \( A \) can be designed for example with gradient method. The cells except the two rows at the edges are identical. A typical template \( A \) is the following:

\[
A = \begin{pmatrix}
0 & 0 & -2 & 0 & 0 \\
0 & -4 & 16 & -4 & 0 \\
-2 & 16 & -39 & 16 & -2 \\
0 & -4 & 16 & -4 & 0 \\
0 & 0 & -2 & 0 & 0
\end{pmatrix}
\]  

Suppose, we have a single layer cellular neural network. If we know the altitude of the surface at grid point \((i,j)\) we fill this altitude into the state of the cell \((i,j)\) and we don't allow the changing of the cell's state. If we don't know the altitude at point \((i,j)\) we fill zero into the inner state of the cell. Then we start the network transient, and at the settled state we get the fitted surface as the output values.

For example, if we have three points in the space then the network of 40x40 cells fits a flat surface on them. Figure 3 shows the start state, and the result after 150 \( \tau \). After 17550 \( \tau \) practically we get the flat surface. (\( \tau \) is the time constant of the cell.)

Figure 3.
The convergence time depends on the given point density. In this example the density was too low. The network was tried with a wedding cake shape. The known point density was 30% and the settling time only 16.5 μ.

The second solution of the fitting problem is approximation. In this case we require only that the surface should approximately fit the known data. In many cases there are wrong points among the given points. If we use interpolation then one wrong point can deform the whole shape, because the surface will fit exactly the given points. If we use approximation then the wrong points don’t disturb the result very much. Now we also have an appropriate potential energy function that is the sum of two parts:

$$E = \int \left( f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2 \right) dx dy + w \sum (f(x_i, y_i) - z_i)^2,$$  \hspace{1cm} (2-3)

where $x_i, y_i, z_i$ are the coordinates of given points.

The smaller the first part is the smoother the examined surface is. The smaller the second part is the better the fitting on the given points will be. The weight factor $w$ determines the relative importance of the two features.

The approximating network differs from the first network only at cells that belongs to given points. At these cells the middle element of matrix $A$ is less by $2w$, than in the first problem. The current $I$ is $2w z_i$, where $z_i$ is the altitude of the given point. The changing of cell state is allowed.

Fig. 4 shows the results using different weight factors. In the second case the weight of fitting is too low, so the result is not proper.

![Figure 4.](image)

3. Door-in-a-floor detection problem

Given a scene: a floor in a building. The three distinct problems to be solved by our analogic CNN algorithms are as follows:

(i) door-handle recognition: after running the CNN algorithm a black spot will show the position of the handle while making anything else to disappear from the scene.

(ii) sign recognition: black rectangles will show the position of supposed signs after the process.

(iii) character extraction: black characters will appear on white background from the original gray-scale image.

Our experiments have shown that even these simple tasks need quite complex analogic CNN algorithms (cca. 20 CNN templates or other transformations for the handle recognition, 15 for the string recognition and 10 for the character extraction), and still the algorithms constructed work if the illumination is fairly uniform. It means we have to add the locally adaptive illumination control of the retina, hence, we combine living and artificial CNN algorithms [5]. The flow diagram for the steps are marked with

(1) BFP.TEM: A template.
(2) THRESH.TEM: Thresholding.
(3) PEELHOR.TEM: Peeling the top.
(4) RESTORE.TEM: Restoring the floor.
(5) SUBTRACT.TEM: Subtracting the floor from the input pixel.
(6) PEELVER.TEM: Peeling the right vertical pixels of the input.
(7-8) FINDRGT.TEM: Finding the right edge pixels of the input.
(9) PEEL.TEM and RESTORE.TEM: Peeling the floor.
(10) SUBTRACT.TEM: Subtracting the floor from the input pixel.
(11) EDGE.TEM: Detecting where the floor ends.
(12) RESTORE.TEM: Restoring the floor.
(13) SUBTRACT.TEM: Subtracting the floor from the input pixel.
(14) HOLEFILL.TEM: Filling the holes.
(15) PEEL.TEM: Peeling the floor.
(16) SKELETON.TEM: Creating a skeleton.
(17) RESTORE.TEM: Restoring the floor.

References


Electronics Research Laboratory, University of California, Berkeley, 1989.
algorithms [5]. The flow diagram of the first algorithm is shown in Fig. 6. The transformation steps are marked with numbers, their meaning is explained below:

1. BFP.TEM: A template that works as a band pass filter.
2. THRESH.TEM: Thresholds one picture by an other.
3. PEELHOR.TEM: Peels black pixels in horizontal direction.
4. RESTORE.TEM: restoring pixels of the input connected with the initial state.
5. SUBTRACT.TEM: Leaves pixels of the initial state darker than the corresponding input pixel.
6. PEELVER.TEM and RESTORE.TEM: See above.
7-8. FINDRIGT.TEM and RESTRICT.TEM: Finds black objects to the right from black pixels of the input.
9. PEEL.TEM and RESTORE.TEM: Deletes small objects.
10. SUBTRACT.TEM: See above.
11. EDGE.TEM: Extracts edges.
12. RESTORE.TEM: See above.
13. SUBTRACT.TEM: See above.
14. HOLEFILL.TEM: Fills holes.
15. PEEL.TEM: See above.
16. SKELETON.TEM: Leaves the skeleton of the objects.
17. RESTORE.TEM: See above.

References
Abstract - This paper presents several different types of median, rank order filters, by changing the template.

1 Introduction

Order statistic filters are a class of filters that are formed by a function of the local sample values. Let $x_1, x_2, \ldots, x_n$ be the local sample values. Then $x(i)$ is called the $i$th order statistic, which is the $i$th largest value in the output of the filter.

$$y_i = \text{med}(x_{i-W/2}, \ldots, x_{i+W/2})$$

where $W = 2v + 1$ is the window size, $v$ is the number of samples. These filters have several advantages over conventional filters. They can remove noise if the impulses are not isolated, and they are capable of preserving the sharp edges of objects. However, they can also blur these transitions.

This paper introduces a new CNN template for implementing the median filter. The paper describes a CNN template that can be used for the implementation of order statistic filters. The CNN template uses the input data values. In contrast, conventional filters require an array with $N$ input data values. The CNN template can also compute the filter output using nonlinear CNN architectures for median filters.

For templates in the form of $w$, the initial state at each cell is given by:

$$\frac{\partial v_x}{\partial t} = \sum_{i \in W} b_i F_y$$

where $F_y$ is the input function.