Multipartite entanglement and its experimental detection

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Outline

1 Motivation
   - Why many-body entanglement is important?

2 Different types of multipartite entanglement
   - Two qubits
   - The three-qubit case in detail
   - Multipartite entanglement

3 Systems with few particles
   - Physical systems
   - Interesting quantum states
   - Witness design
   - Experiment

4 Systems with very many particles
   - Physical systems
   - Spin squeezing
   - Generalized spin squeezing
   - An experiment
Why is multipartite entanglement interesting?

- There have been many experiments recently aiming to create many-body entangled states.
- Quantum Information Science can help to find good targets for such experiments.
- Multipartite entangled states are needed in applications such as metrology.
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Two qubits

Fact

*Remember: There is only a single type of two-qubit entanglement.*

- From many copies of entangled states, we can always distill a singlet using Local Operations and Classical Communication (LOCC).

- From any entangled two-qubit state, we can get to any other entangled two-qubit state through Stochastic Local Operations and Classical Communication (SLOCC).

That is, for any entangled $|\Psi\rangle$ and $|\Phi\rangle$, there are invertible $A$ and $B$ such that

$$|\psi\rangle = A \otimes B |\phi\rangle.$$  

Note that $A$ and $B$ do not have to be Hermitian.
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Three-qubit pure states

- $|\Psi\rangle$ and $|\Phi\rangle$ are equivalent under SLOCC if there are invertible $A$, $B$, and $C$ such that
  
  $$|\Psi\rangle = A \otimes B \otimes C|\Phi\rangle.$$ 

Four classes of states without genuine multipartite entanglement:

Class #1: three-qubit pure product state $|\Psi\rangle_1 \otimes |\Psi\rangle_2 \otimes |\Psi\rangle_3$

Class #2: biseparable states of the type $|\Psi\rangle_1 \otimes |\Psi\rangle_{2,3}$, and $|\Psi\rangle_{2,3}$ is entangled

Class #3: biseparable states of the type $|\Psi\rangle_{1,2} \otimes |\Psi\rangle_3$, and $|\Psi\rangle_{1,2}$ is entangled

Class #4: biseparable states of the type $|\Psi\rangle_{1,3} \otimes |\Psi\rangle_2$, and $|\Psi\rangle_{1,3}$ is entangled
Two classes of genuine multipartite entanglement for pure states:

Class #5: W-class
The W state is defined as

$$|W\rangle = \frac{1}{\sqrt{3}} (|100\rangle + |010\rangle + |001\rangle).$$

Class #6: GHZ-class
The Greenberger-Horne-Zeilinger (GHZ) state is defined as

$$|GHZ\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle).$$

A (GHZ $\cup$ W $\cup$ Bisep $\cup$ Sep)-class state, with an appropriate choice of basis states $|0\rangle$ and $|1\rangle$, has the general form

$$|\Psi_{GHZ}\rangle = \lambda_0|000\rangle + \lambda_1 e^{i\theta}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle,$$

with $\lambda_k \geq 0$.

A (W $\cup$ Bisep $\cup$ Sep)-class state has the form

$$|\Psi_W\rangle = \lambda_0|000\rangle + \lambda_1|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle.$$

Thus, the (GHZ $\cup$ W $\cup$ Bisep $\cup$ Sep) class is the set of all physical states and the (W $\cup$ Bisep $\cup$ Sep)-class is a part of it.
Three-qubit pure states IV

Question:
Why are there two classes of genuine multipartite entanglement?

Answer:
The minimum number of product terms needed for a decomposition is

- 3 for W-class states and
- 2 for GHZ-class states.

This number cannot change by LOCC.

Three-qubit mixed states

Six classes:

Class #1: fully separable states \( \sum_k p_k \rho_1^{(k)} \otimes \rho_2^{(k)} \otimes \rho_3^{(k)} \)

Class #2: (1)(23) biseparable states \( \sum_k p_k \rho_1^{(k)} \otimes \rho_{23}^{(k)} \), not in Class #1

Class #3: (12)(3) biseparable states \( \sum_k p_k \rho_{12}^{(k)} \otimes \rho_3^{(k)} \), not in Class #1

Class #4: (13)(2) biseparable states \( \sum_k p_k \rho_{13}^{(k)} \otimes \rho_2^{(k)} \), not in Class #1

Class #5: W-class states:
mxtr of pure \((W \cup \text{Bisep} \cup \text{Sep})\)-class states, not in Classes #1-4

Class #6: GHZ-class states: mxtr of pure \((\text{GHZ} \cup W \cup \text{Bisep} \cup \text{Sep})\)-class states, not in Classes #1-5

Biseparable states: mixture of states of classes #2, #3 and #4.
The extension of the classification of pure states to mixed states leads to convex sets:

Witnesses for GHZ and W-class states

Entanglement witnesses for detecting states of a given class:

GHZ-class states

\[ W_{\text{GHZ}}^{(P)} := \frac{3}{4} \mathbb{1} - |\text{GHZ}\rangle\langle \text{GHZ}|. \]

W-class states

\[ W_{\text{W}}^{(P)} := \frac{2}{3} \mathbb{1} - |\text{W}\rangle\langle \text{W}|. \]

\[ W_{\text{GHZ}}^{(P)} := \frac{1}{2} \mathbb{1} - |\text{GHZ}\rangle\langle \text{GHZ}|. \]

There are states that are biseparable with respect to all the three bipartitions, but they are not fully separable.

\[ \rho = \sum_k p_k \rho^{(k)}_{12} \otimes \rho^{(k)}_3 \]

\[ \rho = \sum_k p'_k \rho^{(k)}_1 \otimes \rho^{(k)}_{23} \]

\[ \rho = F_{12} \sum_k p''_k \rho^{(k)}_2 \otimes \rho^{(k)}_{13} F_{12} \]
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4 qubits: There are 9 families and infinite number of SLOCC equivalence classes.

For many qubits, the practically meaningful classification is
- (Fully) separable
- Biseparable entangled
- Genuine multipartite entangled
A state is (fully) separable if it can be written as
\[ \sum_k p_k \rho_1^{(k)} \otimes \rho_2^{(k)} \otimes \ldots \otimes \rho_N^{(k)}. \]

A pure multi-qubit quantum state is called biseparable if it can be written as the tensor product of two multi-qubit states
\[ |\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle. \]

Here \(|\psi\rangle\) is an \(N\)-qubit state. A mixed state is called biseparable, if it can be obtained by mixing pure biseparable states.

If a state is not biseparable then it is called genuine multi-partite entangled.
Convex sets for the multipartite case

- The idea of convex sets also work for the multi-qubit case: A state is biseparable if it can be obtained by mixing pure biseparable states.
Examples

Two entangled states of four qubits:

\[ |GHZ_4\rangle = \frac{1}{\sqrt{2}} (|0000\rangle + |1111\rangle), \]

\[ |\Psi_B\rangle = \frac{1}{\sqrt{2}} (|0000\rangle + |0011\rangle) = \frac{1}{\sqrt{2}} |00\rangle \otimes (|00\rangle + |11\rangle). \]

- The first state is genuine multipartite entangled, the second state is biseparable.
Other possible definition of genuine multipartite entanglement

- Alternative definition: a state is genuine multipartite entangled if it is inseparable with respect to all bipartitions.

**Example**

Mixture of the two biseparable states (chains of singlets)

<table>
<thead>
<tr>
<th>50%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Diagram of biseparable states" /></td>
<td><img src="image.png" alt="Diagram of biseparable states" /></td>
</tr>
</tbody>
</table>

It is inseparable with respect to all bipartitions.

- This state can be created in a two-qubit experiment.
Geometric measure of entanglement

- There is not a single entanglement measure for multipartite systems.

**Talk by JENS SIEWERT**

**Definition**

For pure states, the geometric measure of entanglement is defined as

$$E_{\sin^2}(|\Psi\rangle) = 1 - \left( \max_{|\Psi_P\rangle \in \text{PRODUCT}} \langle \Psi | \Psi_P \rangle \right)^2.$$ 

For mixed states, it is defined by a convex roof construction

$$E_{\sin^2}(\rho) = \min_{\{ |\Psi_k\rangle, p_k \} : \rho = \sum_k p_k |\Psi_k\rangle \langle \Psi_k |} \sum_k p_k E_{\sin^2}(|\Psi_k\rangle).$$

- It is possible to calculate it for some pure states and for some mixed states.

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Physical systems

State-of-the-art in experiments

- 8 qubits with trapped cold ions (Nature, 2005)
- 10 qubits with photons (Nature Physics, 2010)

Main Challenges

- How to obtain useful information when only \textit{local} measurements are possible?

\textit{In principle, the entanglement witness method has the advantage that only one observable, the entanglement witness, needs to be measured. In practice, the measurement of this observable may be done by a series of local measurements. ... At this point the advantage over basic state tomography becomes somewhat questionable.} (B. TERHAL, IBM Watson Research Center, 2002)
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States

Quantum states in experiments:

- Greenberger-Horn-Zeilinger (GHZ) state or "Schrödinger cat state"
  \[
  |\psi\rangle = |0\rangle_{1}|1\rangle_{2}|1\rangle_{3} + |1\rangle_{1}|0\rangle_{2}|0\rangle_{3}.
  \]

- Cluster state, graph state (obtained in Ising spin chains)
  \[
  |\psi\rangle = |\sigma_z^{(1)}\sigma_z^{(2)}\rangle |\sigma_z^{(2)}\sigma_z^{(3)}\rangle |\sigma_z^{(3)}\sigma_z^{(4)}\rangle.
  \]

- Symmetric Dicke states
  \[
  |\psi\rangle = |0\rangle_{1}|0\rangle_{2}|0\rangle_{3} + |1\rangle_{1}|1\rangle_{2}|1\rangle_{3} + |2\rangle_{1}|2\rangle_{2}|2\rangle_{3} + \ldots
  \]

- Singlet states
  \[
  (\Delta J_j)^2 = 0 \quad \text{for} \; j = x, y, z.
  \]
Aims when designing a witness

Definition

An entanglement witness $W$ is an operator that is positive on all separable (biseparable) states. Thus, $\text{Tr}(W\rho) < 0$ signals entanglement (genuine multipartite entanglement). Horodecki 1996; Terhal 2000; Lewenstein, Kraus, Cirac, Horodecki 2002

There are two main goals when searching for entanglement witnesses:

- Large robustness to noise
- Few measurements
Robustness to noise

A state mixed with white noise is given as

$\varrho(p_{\text{noise}}) = (1 - p_{\text{noise}})\varrho + p_{\text{noise}}\varrho_{\text{noise}}$

where $p_{\text{noise}}$ is the ratio of noise and $\varrho_{\text{noise}}$ is the noise. If we consider white noise then $\varrho_{\text{noise}} = \mathbb{1}/2^N$.

Definition

The noise tolerance of a witness $\mathcal{W}$ is characterized by the largest $p_{\text{noise}}$ for which we still have

$$\text{Tr}(\mathcal{W}_\varrho) < 0.$$
Only local measurements are possible

Definition

A single **local measurement setting** is the basic unit of experimental effort.

A local setting means measuring operator $A^{(k)}$ at qubit $k$ for all qubits.

- All two-qubit, three-qubit correlations, etc. can be obtained.

\[
\langle A^{(1)} A^{(2)} \rangle, \langle A^{(1)} A^{(3)} \rangle, \langle A^{(1)} A^{(2)} A^{(3)} \rangle, \ldots
\]
Decomposition of an operator

- All operators must be decomposed into the sum of locally measurable terms and these terms must be measured individually.
- For example,

\[
|GHZ_3\rangle\langle GHZ_3| = \frac{1}{8}(\mathbb{1} + \sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_z^{(3)} + \sigma_z^{(2)}\sigma_z^{(3)})
\]

\[
+ \frac{1}{4}\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}
\]

\[
- \frac{1}{16}(\sigma_x^{(1)} + \sigma_y^{(1)})(\sigma_x^{(2)} + \sigma_y^{(2)})(\sigma_x^{(3)} + \sigma_y^{(3)})
\]

\[
- \frac{1}{16}(\sigma_x^{(1)} - \sigma_y^{(1)})(\sigma_x^{(2)} - \sigma_y^{(2)})(\sigma_x^{(3)} - \sigma_y^{(3)}).
\]


- As \( N \) increases, the number of terms increases exponentially for projectors to quantum pure states.
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Basic methods for designing witnesses

Three methods for designing witnesses:

- Projector witness, i.e., witness defined with the projector to a highly entangled quantum state

- Witness based on the projector witness

- Witness independent of the projector witness
A witness detecting genuine multi-qubit entanglement in the vicinity of a pure state $|\Psi\rangle$ is

$$\mathcal{W}_\Psi^{(P)} := \lambda_\Psi^2 \mathbb{1} - |\Psi\rangle\langle \Psi|,$$

where $\lambda$ is the maximum of the Schmidt coefficients for $|\Psi\rangle$, when all bipartitions are considered.


A symmetric witness operator can always be decomposed as

$$P = \sum c_k A_k \otimes A_k \otimes A_k \otimes ... \otimes A_k.$$

For symmetric operators, the number of settings needed is increasing **polynomially** with the number of qubits.

GHZ states (robustness to noise is $\frac{1}{2}$ for large $N$!)

$$\mathcal{W}_{\text{GHZ}}^{(P)} := \frac{1}{2} \mathbb{1} - |\text{GHZ}_N\rangle\langle \text{GHZ}_N|.$$  

Cluster states

$$\mathcal{W}_{\text{CL}}^{(P)} := \frac{1}{2} \mathbb{1} - |\text{CL}_N\rangle\langle \text{CL}_N|.$$  

Dicke state

$$\mathcal{W}_{\text{D}(N,N/2)}^{(P)} := \frac{1}{2} \frac{N}{N-1} \mathbb{1} - |\text{D}_{N}^{(N/2)}\rangle\langle \text{D}_{N}^{(N/2)}|.$$  

W-state

$$\mathcal{W}_{W}^{(P)} := \frac{N-1}{N} \mathbb{1} - |\text{D}_{N}^{(1)}\rangle\langle \text{D}_{N}^{(1)}|.$$
Witnesses based on the projector witness

- We construct witnesses that are easier to measure than the projector witness.

- Idea: If $W^{(P)}$ is the projector witness and

$$W - \alpha W^{(P)} \geq 0$$

is fulfilled for some $\alpha > 0$, then $W$ is also a witness.

Witnesses based on the projector witness II

Example
Witness requiring only two measurement settings for GHZ states

\[ \mathcal{W}_\text{GHZ}^{(P)} := \frac{1}{2} \mathbb{1} - \left| \text{GHZ}_N \right\rangle \left\langle \text{GHZ}_N \right| \]

\[ \leq \mathcal{W}_\text{GHZ}^{(P2)} := \mathbb{1} - \frac{1}{2} X_1 X_2 X_3 \ldots X_N - \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 1 \end{bmatrix} \]

Measurement settings ⇒ \[ [X \ X \ X \ X \ \ldots] \quad [Z \ Z \ Z \ Z \ \ldots] \]

- Any state detected by \( \mathcal{W}_\text{GHZ}^{(P2)} \) is also detected by \( \mathcal{W}_\text{GHZ}^{(P)} \).

Witnesses independent from the projector witness

- Witnesses without any relation to the projector witness.

- With an easily measurable operator $M$, we make a witness of the form

$$\mathcal{W} := c \mathbb{1} - M,$$

where $c$ is some constant.

- We have to set $c$ to

$$c = \max_{|\psi\rangle \in \mathcal{B}} \langle M | \psi \rangle,$$

where $\mathcal{B}$ is the set of biseparable states. This problem is typically hard to solve.
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An experiment: Cluster state with photons

Experiment for creating a four-photon cluster state (Weinfurter group, 2005)
An experiment: Dicke state with photons

A photo of a real experiment (six-photon Dicke state, Weinfurter group, 2009):
Experiment: W-state with ions

- Experimental observation of an 8-qubit W-state with trapped ions.

Quantum state tomography

- The density matrix can be reconstructed from $3^N$ measurement settings.

- The measurements are

  1. XXXX
  2. XXXY
  3. XXXZ
  ...
  $3^4$. ZZZZ

- Note again that the number of measurements scales exponentially in $N$. 
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Physical systems

State-of-the-art in experiments

- 100,000 atoms realizing an array of 1D Ising spin chains (Nature, 2003)
- Spin squeezing with $10^6 - 10^{12}$ atoms (Nature, 2001)

Main challenge

- The particles cannot be addressed individually.
- Only collective quantities can be measured.
- New type of entangled states and entanglement criteria are needed.
Many-particle systems

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

  \[ J_l := \frac{1}{2} \sum_{k=1}^{N} \sigma_l^{(k)}, \]

  where \( l = x, y, z \) and \( \sigma_l^{(k)} \) a Pauli spin matrices.

- We can also measure the

  \[ (\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2 \]

  variances.
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Spin squeezing

Definition

Uncertainty relation for the spin coordinates

$$(\Delta J_x)^2 (\Delta J_y)^2 \geq \frac{1}{4} |\langle J_z \rangle|^2.$$ 

If $(\Delta J_x)^2$ is smaller than the standard quantum limit $\frac{1}{2} |\langle J_z \rangle|$ then the state is called spin squeezed (mean spin in the $z$ direction!).


Talk by ALICE SINATRA
Spin squeezing II

**Definition**

Spin squeezing criterion for the detection of quantum entanglement

\[
\frac{(\Delta J_x)^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} \geq \frac{1}{N}.
\]

If a quantum state violates this criterion then it is entangled.

- Application: Quantum metrology, magnetometry.

[ A. Sørensen *et al.*, Nature **409**, 63 (2001) ]

Talk by AGOSTO SMERZI
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Complete set of the generalized spin squeezing criteria

Let us assume that for a system we know only
\[ \vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle), \]
\[ \vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle). \]

Then any state violating the following inequalities is entangled
\[ \langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq N(N + 2)/4, \]
\[ (\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq N/2, \]
\[ \langle J_{k}^2 \rangle + \langle J_{l}^2 \rangle - N/2 \leq (N - 1)(\Delta J_m)^2, \]
\[ (N - 1) \left[ (\Delta J_k)^2 + (\Delta J_l)^2 \right] \geq \langle J_{m}^2 \rangle + N(N - 2)/4. \]

where \( k, l, m \) takes all the possible permutations of \( x, y, z \).

The polytope

- The previous inequalities, *for fixed* \( \langle J_{x/y/z} \rangle \), describe a polytope in the \( \langle J^2_{x/y/z} \rangle \) space.

- Separable states correspond to points inside the polytope. Note: Convexity comes up again!
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The physical system

- Rb atoms + light

Diagram:
- Laser
- Feedback
- Measurement
Atoms interact with light. The light is measured, projecting the atoms into a squeezed state.

Room temperature experiments: $10^{12}$ atoms


- Vapor cells

Cold atom experiments: $10^6$ atoms.

- Laser cooling, sample in an optical dipole trap.
- Atoms are transferred from a MOT to a dipole trap.
An experiment

Spin squeezing in a cold atomic ensemble (not BEC!)

Picture from M.W. Mitchell, ICFO, Barcelona.
Summary

We discussed entanglement detection in multipartite systems.

We considered

- systems with few particles in which the particles could be individually addressed.
- systems with very many particles, without the possibility of individual addressing


THANK YOU FOR YOUR ATTENTION!