Fortschr. Phys. **49** (2001) 10–11, 993–1000

Coherent-state Approach to Entanglement and Teleportation

J. JANSZKY[1]), A. GÁBRIS[1]), M. KONIORCZYK[1,2], A. VUKICS[1]), and P. ADAM[1,2])

[1]) Department of Nonlinear and Quantum Optics,
Research Institute for Solid State Physics and Optics,
Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary
[2]) Institute of Physics, University of Pécs, Ifjúság út 6. H-7624 Pécs, Hungary

Abstract

Two-mode oscillator quantum states are described via coherent state superpositions. The one-complex-plane coherent-state representation is introduced as a generalization of low-dimensional coherent-state representations of single mode fields. Application to two-mode squeezed states is emphasized, because of their entangled nature and applicability for quantum communication. Continuous variable quantum teleportation is treated in this framework.

PACS: 03.67.Hk, 03.65.Ud, 42.50.Dv

1. Introduction

Squeezing may be regarded as one of the most interesting phenomena in the quantum optics of single mode fields [1, 2]. Squeezed states are nowadays typical textbook-examples to demonstrate the application of the arsenal of theoretical quantum optics for description of the state of a single mode fields [3, 4]. One of these methods is the low-dimensional coherent-state representation of states, namely quadrature-squeezed states can be expressed as superpositions of coherent states along straight lines of the phase space [5]. This approach has shown to be useful in describing other states of a single mode field as well. As coherent states form an overcomplete basis, states may be expressed as superpositions of coherent states in lower dimensional [6, 7] or even discrete [8] subsets of the phase space.

Because of the intensively growing interest in entangled states, especially due to fundamental questions of quantum mechanics, and the wide range of applications in the field of quantum computation and quantum information, the attention has turned to multimode fields. Both the generation of squeezed states and entangled states in optics is related to coherent generation of photon pairs in nonlinear optical phenomena [9–11]. These kind of processes are amongst the most widely used sources of entangled pairs in experiments [12]. Squeezing and entanglement are related concepts. But can we take advantage of the theoretical methods developed for single mode fields in this context?

In this paper, we shall present a method of representing quadrature-entangled states of two-mode light fields by the means of coherent states. The set of states possibly represented in this way is not limited to entangled states, but it covers the entire Hilbert space. The representation presented here is constructed in a manner very similar to the one-dimensional coherent-state representations [5]. Instead of a superposition along a one-dimensional curve in the complex phase space, the representation for two modes shall employ an integral on a single complex (i.e. two real dimensional) plane inside the two complex (four real) dimensional phase space.

 0015-8208/01/10-1111-0993 $ 17.50+.50/0

To demonstrate the capabilities of this kind of representation, we shall formulate a description of continuous variable teleportation [13, 14], which is one of the most typical applications of entanglement. The first quantum optical formulation of Braunstein and Kimble in Ref. [15] utilizes the Wigner-function formalism. Their scheme may also be described in terms of either wavefunctions on a quadrature-state basis [16, 17] or Fock-states [17, 18]. A general covariant description in terms of arbitrary canonically conjugate observables and their eigenstates is also possible [19]. Our approach offers a useful alternative for certain cases, especially if some of the states involved obey a simple analytic representation. Description of the entangled basis corresponding to the Bell-state measurement reveals another possibility of constructing one-complex-plane representations.

This paper is organized as follows: In section 2 we shall briefly summarize the one-dimensional coherent-state representations, focusing mainly on the straight-line superpositions. In section 3 we shall introduce the one-complex-plane representation used for two-mode states. In section 4 the continuous variable teleportation scheme is described using the representations discussed so far, and introducing yet another construction. Section 5 summarizes our results.

2. One-dimensional Representations

What makes representations with coherent states possible is their well-known property of completeness: every state can be expressed as a superposition of them. In the most general form, for pure states this means the existence of a weight function such that

$$|\psi\rangle = \int_{\boldsymbol{C}} f(\alpha, \alpha^*)\, |\alpha\rangle\, d^2\alpha\,. \tag{1}$$

Coherent states are not linearly independent, therefore there exist a multiple of weight functions for which Eq. (1) holds. Because of these properties, the set of coherent states is often called an over- or supercomplete basis. The possibility of one-dimensional representations originates from this overcompleteness, and is justified by the Cahill theorem [20].

A possible way of obtaining straight-line representations for a single mode field state is to define an orthonormal basis, as done in Ref. [6]. The basis elements are expressed as superpositions of coherent states on a straight line, thus any state can be expressed with them. For example, if we take the real axis for representation, an orthonormal basis can be defined as

$$|h_n\rangle_\gamma := \mathcal{N}_n(\gamma^2) \int_{\boldsymbol{R}} H_n(\mu x)\, e^{-\frac{x^2}{\gamma^2}}\, |x\rangle\, dx \qquad \gamma \in \boldsymbol{R} \tag{2}$$

$$\mu = \sqrt{\frac{2(1+\gamma^2)}{\gamma^2(2+\gamma^2)}}\,,$$

where H_n stands for the n-th order Hermite polynomial, and $\mathcal{N}_n(\gamma^2)$ is a normalization factor, so $\||h_n\rangle_\gamma\|^2 = 1$ holds. Once there is an orthonormal basis, every state can be uniquely expressed with them. In the actual example this means

$$\begin{aligned} |\psi\rangle &= \sum_{n\in\boldsymbol{N}} \langle h_n \mid \psi\rangle\, |h_n\rangle_\gamma \\ &= \sum_{n\in\boldsymbol{N}} \langle h_n \mid \psi\rangle\, \mathcal{N}_n(\gamma^2) \int_{\boldsymbol{R}} H_n(\mu x)\, e^{-\frac{x^2}{\gamma^2}}\, |x\rangle\, dx\,. \end{aligned} \tag{3}$$

In this formula the order of summing and integrating can be interchanged, provided that we extend our search for weight distributions from real functions to distributions:

$$|\psi\rangle = \int_{\boldsymbol{R}} F^{\psi}(x)\, e^{-\frac{x^2}{\gamma^2}}\, |x\rangle\, dx$$
$$F^{\psi}(x) = \sum_{n \in \boldsymbol{N}} \langle h_n \mid \psi\rangle\, \mathcal{N}_n(\gamma^2)\, H_n(\mu x)\,. \tag{4}$$

These representations are best suitable for describing states whose Wigner function does not extend "too far" away from the line of states used for representation. If that is the case, the weight distributions will more likely be regular functions rather than distributions. For instance squeezed states are best represented on a straight line perpendicular to the direction of squeezing. For the horizontally and vertically squeezed vacua it is exactly

$$|\text{horiz. sq. vac.}\rangle = \mathcal{N}(\gamma^2) \int_{\boldsymbol{R}} e^{-\frac{y^2}{\gamma^2}}\, |iy\rangle\, dy\,, \tag{5}$$

$$|\text{vert. sq. vac.}\rangle = \mathcal{N}(\gamma^2) \int_{\boldsymbol{R}} e^{-\frac{x^2}{\gamma^2}}\, |x\rangle\, dx\,, \tag{6}$$

respectively, where $\gamma^2 = e^{2r} - 1$, r being the squeezing parameter.

3. A Two-dimensional Coherent-state Representation

In order to describe entanglement, it is necessary to find a complete representation for the two-mode oscillator Hilbert space. These representations exist, since the two-mode coherent states also form an overcomplete basis for the entire Hilbert space:

$$\frac{1}{\pi^2} = \int_{\boldsymbol{C}^2} |\alpha\rangle\,\langle\alpha| \otimes |\beta\rangle\,\langle\beta|\, d^2\alpha\; d^2\beta = \mathbf{1}. \tag{7}$$

What we need to do is to select the appropriate one, which is the most suitable for our purposes.

Let us follow the same steps as in constructing the straight-line coherent-state representation in the previous section. First we choose a lower dimensional subspace of the phase space with the formula

$$(\alpha, \alpha^*) \in \boldsymbol{C}^2\,. \tag{8}$$

Next, one has to choose a set of weight functions which are appropriate for the definition of a basis. We show, that the Laguerre-2D functions [21] are suitable for this purpose. The Laguerre-2D functions are defined as

$$l_{m,n}(z, z^*) = \frac{1}{\pi}\, e^{-\frac{zz^*}{2}}\, \frac{1}{\sqrt{m!n!}} \sum_{j=0}^{\min\{m,n\}} \frac{m!n!}{j!(m-j)!\,(n-j)!}\, (-1)^j\, z^{m-j} z^{*n-j}\,. \tag{9}$$

They are complete in the sense

$$\sum_{m,\,n\in \boldsymbol{N}_0} l_{m,n}(z,z^*)\, l^*_{m,n}(w,w^*) = \delta(z-w, z^*-w^*), \tag{10}$$

and orthonormal:

$$\int_{\boldsymbol{C}} l^*_{k,l}(z,z^*)\, l_{m,n}(z,z^*)\, d^2z = \delta_{k,m}\delta_{l,n}\,. \tag{11}$$

An orthonormal basis for the Hilbert-space of the two-mode oscillator can be defined after introducing the appropriate normalization factor:

$$\left|l_{m,n}\right\rangle_\gamma = \mathcal{N}_{m,n}(\gamma) \int_{\boldsymbol{C}} l_{m,n}(\mu\alpha,\mu\alpha^*)\, e^{\frac{|\alpha|^2\mu}{2}}\, e^{-\frac{|\alpha|^2}{\gamma^2}}\, |\alpha\rangle\, |\alpha\rangle^*\, d^2\alpha\,, \tag{12}$$

where

$$\mathcal{N}_{m,n}(\gamma) = \frac{\sqrt{1+2\gamma^2}}{\gamma^2\pi} \left(\frac{1+\gamma^2}{\gamma^2}\right)^{\frac{m+n}{2}} \tag{13}$$

$$\mu = \sqrt{\frac{1+2\gamma^2}{\gamma^2(\gamma^2+1)}}\,.$$

With the above definition, the set $\{|l_{m,n}\rangle \mid m,n \in \boldsymbol{N}_0\}$ forms an orthonormal basis in the two-mode Hilbert space. This can be most easily proven by constructing a unitary mapping from some other orthonormal basis to the one under discussion. We have the two-mode squeezing operator and the Fock-states to satisfy these requirements.

The proof can be outlined as follows: Let the squeezing operator $\hat{S}^{(2)}$ defined by describing its effect on the ladder operators:

$$\hat{a}' := u\hat{a} + v\hat{b}^\dagger\,, \qquad u, v \in \boldsymbol{C} \tag{14}$$

$$\hat{b}' := u\hat{b} + v\hat{a}^\dagger\,. \tag{15}$$

Here $|u|^2 - |v|^2 = 1$ ensures the correct commutation relations of the ladder operators $\hat{a}$, $\hat{b}$ etc. of the first and second mode, respectively. Using these notations, we may go on calculating what states the Fock-states are mapped onto, namely:

$$\hat{S}^{(2)}|n,m\rangle = \hat{S}^{(2)}\,\frac{\hat{a}^{\dagger n}}{\sqrt{n!}}\,\frac{\hat{b}^{\dagger m}}{\sqrt{m!}}\,|0,0\rangle$$

$$= \frac{1}{\sqrt{n!m!}}\,(u^*\hat{a}^\dagger + v^*\hat{b})^n\,\left(u^*\hat{b}^+ v^*\hat{a}\right)^m \hat{S}^{(2)}\,|0,0\rangle\,. \tag{16}$$

Concerning $\hat{S}^{(2)}\,|0,0\rangle$ in the last line of this expression, it can be shown after straightforward calculation of the characteristic function $\chi_S(\eta,\xi) = \langle 0,0|\,\hat{S}^{(2)\dagger}\,e^{\eta\hat{a}^\dagger+\eta^*\hat{a}}\,e^{\xi\hat{b}^\dagger+\xi^*\hat{b}}\,\hat{S}^{(2)}\,|0,0\rangle$, that it leads exactly to the same result as $\chi_L(\eta,\xi) = \langle l_{0,0}|_\gamma\,e^{\eta\hat{a}^\dagger+\eta^*\hat{a}}\,e^{\xi\hat{b}^\dagger+\xi^*\hat{b}}\,\left|l_{0,0}\right\rangle_\gamma$ with

$$\gamma := \sqrt{\frac{v}{u-v}}\,, \tag{17}$$

leading to the identity

$$\hat{S}^{(2)}\,|n,m\rangle = \frac{1}{\sqrt{n!m!}}\,(u^*\hat{a}^\dagger + v^*\hat{b})^n\,(u^*\hat{b}^\dagger + v^*\hat{a})^m\,\mathcal{N}(\gamma)\int_{C} e^{-\frac{|\alpha|^2}{\gamma^2}}\,|\alpha\rangle\,|\alpha\rangle^*\,d^2\alpha\,. \quad (18)$$

The right hand side of (17) can be evaluated by replacing the annihilation operators by the appropriate eigenvalues and vice-versa, and also by the

$$\int_{C} e^{-\frac{|\alpha|^2}{\gamma^2}}\,\hat{a}^n\,|\alpha\rangle\,\left|\alpha^*\right\rangle d^2\alpha = \int_{C} e^{-\frac{|\alpha|^2}{\gamma^2}}\left(\frac{u}{v}\,\alpha^*\right)^n |\alpha\rangle\,|\alpha\rangle^*\,d^2\alpha$$

$$\int_{C} e^{-\frac{|\alpha|^2}{\gamma^2}}\,\hat{b}^n\,|\alpha\rangle\,\left|\alpha^*\right\rangle d^2\alpha = \int_{C} e^{-\frac{|\alpha|^2}{\gamma^2}}\left(\frac{u}{v}\,\alpha\right)^n |\alpha\rangle\,|\alpha\rangle^*\,d^2\alpha \quad (19)$$

relations for the creation operators. These relations can be verified for example, by expanding both sides of the equations using the Fock-state basis. By repeated application of the eigenvalue equations, the (19) identities and the commutation relations, one can see that

$$\hat{S}^{(2)}\,|n,m\rangle = \left|l_{m,n}\right\rangle_\gamma\,. \quad (20)$$

Thus we have found a one-complex-plane representation for the basis states by applying the two-mode squeezing operator on Fock-states. We can also conclude, that these states form an orthonormal basis on the Hilbert space of the two modes. This basis is built up by entangled states.

4. Reformulation of Continuous Variable Teleportation

In this section we describe quantum teleportation of continuous variables, partly as an application of results in the previous section, and also as a slightly different application of one-dimensional representations in this field.

In the teleportation scheme of Kimble et al., Alice, the sender, and Bob, the receiver share an entangled (EPR) state $|\Psi^\gamma_{\mathrm{EPR}}\rangle_{23}$ in modes 2 and 3, which is a two-mode squeezed vacuum with squeezing parameter γ. Alice has mode 1 in the unknown state $|\Psi_{\mathrm{in}}\rangle$, and carries out a joint measurement on modes 1 and 2. The states this measurement projects onto are maximally entangled states of modes 1 and 2, namely the so called quadrature Bell-states. The measurement result is sent to Bob through a classical channel, who carries out a unitary transformation on the output state to recover the teleported state in mode 3.

According to the scheme outlined above, our task is now to describe the EPR state and the projective measurement in our formulation. As regards of the EPR pair, according to Eqs. (23) and (14) we may simply write

$$|\Psi^\gamma_{\mathrm{EPR}}\rangle_{23} = \left|l_{0,0}\right\rangle = \mathcal{N}_{0,0}\int_{C} e^{-\frac{2|\alpha|^2}{\gamma^2}}\,|\alpha\rangle\,|\alpha\rangle^*\,d^2\alpha\,, \quad (21)$$

where we have substituted parameter γ with $\gamma/\sqrt{2}$ for convenience. An ideal entangled state is obtained in the infinite squeezing limit $\gamma \to \infty$, with $e^{-\frac{2|\alpha|^2}{\gamma^2}} \to 1$ in that case.

Now let us describe the projective measurement at Alice's side. Alice combines modes 1 and 2 on a 50-50% beam-splitter, and then measures quadrature $\hat{x}$ of mode 1, and quadrature $\hat{p}$ of mode 2. The detectors are supposed to be ideal, that is, they project onto eigenstates of the quadrature operators. Thus we have to describe these eigenstates first. A quadrature eigenstate with eigenvalue 0 is an infinitely squeezed one-mode vacuum: it can be obtained from Eqs. (5) and (6) in the infinite squeezing limit (We denote quadrature-eigenstates by $||\ldots\rangle$):

$$\begin{aligned} ||P=0\rangle &= \lim_{\gamma\to\infty} \int_{\boldsymbol{R}} dx\, e^{-\frac{x^2}{\gamma^2}} |x\rangle = \int_{\boldsymbol{R}} dx\, |x\rangle \\ ||X=0\rangle &= \lim_{\gamma\to\infty} \int_{\boldsymbol{R}} dy\, e^{-\frac{y^2}{\gamma^2}} |iy\rangle = \int_{\boldsymbol{R}} dy\, |iy\rangle\,. \end{aligned} \tag{22}$$

As a matter of fact, these are superpositions of all coherent states along one of the axes of the phase space with equal weight, therefore they are not normalizable. This is a usual property of "position" and "momentum" eigenstates. We have therefore omitted unnecessary normalization factors. Quadrature eigenstates for arbitrary value of quadrature are then obtained by shifting states in Eq. (22) applying the Glauber displacement operator $\hat{D}(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$:

$$\begin{aligned} ||P\rangle &= \hat{D}(iP)\,||P=0\rangle = \int_{\boldsymbol{R}} dx\, e^{ixP}\, |x+iP\rangle \\ ||X\rangle &= \hat{D}(X)\,||X=0\rangle = \int_{\boldsymbol{R}} dy\, e^{-iXy} |X+iy\rangle\,. \end{aligned} \tag{23}$$

The projective measurement by the quadrature detectors projects onto the state $||X\rangle_1\,||P\rangle_2$. As a result of the beam-splitter transformation, it is converted to a quadrature Bell-state, which is an eigenstate of the joint observable of Alice's measurement. This can be simply calculated from Eq. (23). Coherent states interfere on beam-splitters as classical amplitudes. We choose the beam-splitter parameters so that the inverse transform of arbitrary superposition $\int_{\boldsymbol{C}} d^2\alpha \int_{\boldsymbol{C}} d^2\beta\, \Phi(\alpha,\beta)\, |\alpha\rangle_1\, |\beta\rangle_2$ of coherent states can be written as:

$$\int_{\boldsymbol{C}} d^2\alpha \int_{\boldsymbol{C}} d^2\beta\, \Phi(\alpha,\beta) \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle_1 \left|\frac{\beta-\alpha}{\sqrt{2}}\right\rangle_2 . \tag{24}$$

Applying this to $||X\rangle_1\,||P\rangle_2$ in the form of Eq. (26), we can write

$$|\Psi^{(A)}_{\text{Bell}}\rangle = \int_{\boldsymbol{C}} d^2\alpha\; e^{A\alpha^* - A^*\alpha}\, |\alpha + A\rangle_1\, |\alpha^* - A^*\rangle_2\,, \tag{25}$$

where we have introduced the new variables

$$\alpha := \frac{x+iy}{\sqrt{2}}\,, \qquad A := \frac{X+iP}{\sqrt{2}}\,. \tag{26}$$

Here A is the result of the measurement, which is communicated to Bob in the scheme. Note, that we have obtained a one-complex-plane representation described in Section 2 of quadrature Bell-states relevant in the description of teleportation with a different, but analog method: instead of squeezing Fock-states, we have applied a beam-splitter transformation to the orthogonal basis $||X\rangle\,||P\rangle$, thereby obtaining a maximally entangled basis with elements $\{|\Psi^{(A)}_{\text{Bell}}\rangle|\, A \in \boldsymbol{C}\}$ on the product Hilbert space describing the two-mode field. For $A = 0$ we have the infinitely squeezed vacuum state.

Now we are ready to evaluate the result of the projective measurement. We write the state to be teleported in the Glauber analytic representation:

$$|\Psi_{\text{in}}\rangle_1 = \int_{C} d^2\alpha \, e^{-\frac{|\alpha|^2}{2}} f(\alpha^*) \, |\alpha\rangle_1 \,, \tag{27}$$

the state is described by the function f. The joint state of the three modes is

$$|\Psi_{\text{joint}}\rangle_{123} = |\Psi_{\text{in}}\rangle_1 \, |\Psi^{\gamma}_{\text{EPR}}\rangle_{23} \,. \tag{28}$$

The state of the system after the measurement resulting the eigenvalue A is expressed by the partial inner product

$$|\Psi_f\rangle_3 =_{12} \langle \Psi^{(A)}_{\text{Bell}} \mid \Psi_{\text{joint}}\rangle_{123} . \tag{29}$$

Substituting Eqs. (25) and (27) into (29), we obtain an expression containing two complex integrals. It is shown in detail in Ref. [22] that this can be evaluated and yields

$$|\Psi_f\rangle_3 = \int_{C} d^2\alpha \, e^{-\frac{2|\alpha|^2}{\gamma^2}} \, \hat{D}(-2A) \, e^{-\frac{|\alpha|^2}{2}} f(\alpha^*) \, |\alpha\rangle_3 \,. \tag{30}$$

In the infinite squeezing limit $\gamma \to \infty$, the Gaussian factor $e^{-\frac{2|\alpha|^2}{\gamma^2}}$ tends towards 1, thus it can be seen, that Bob has to apply the unitary transformation $\hat{D}(2A)$ in mode 3 to obtain the teleported state. This is the result expected. In the physically realistic case of finite squeezing, Bob may also apply the appropriate shift, which yields

$$|\Psi_{\text{teleported}}\rangle = \int_{C} d^2\alpha \, e^{-\frac{2|\alpha - 2A|^2}{\gamma^2}} \, e^{-\frac{|\alpha|^2}{2}} f(\alpha^*) \, |\alpha\rangle_3 \,. \tag{31}$$

The teleportation is imperfect in this case: a Gaussian smoothing factor appears, which is approximately unity for large squeezing parameter, and for α values close to A. This is a consequence of finite number of photons involved in the entangled state. Continuous variable quantum teleportation cannot be perfect in reality.

5. Conclusion

We have adopted a coherent-state approach to the theory of two-mode fields, with special emphasis on the description of entanglement and teleportation. We have shown how an orthonormal basis can be constructed as a superposition of two-mode coherent states by integrating on one single complex plane instead of two of them. This basis is obtained by applying the two-mode squeezing operator to Fock states. Another basis was constructed by applying a beam-splitter transformation onto product quadrature states, namely the quadrature Bell-basis. The process of continuous variable quantum teleportation was constructed in this formulation.

The formalism developed here is applicable for analysis of optical schemes involving continuous variable entanglement, because of the known advantageous properties of coherent states. Teleportation with losses for instance can be treated by introducing additional beam-splitters to the scheme, and density matrices can be treated in the R-representation in that case. The formulation is particularly useful for analysis of states which can be easily constructed as superposition of multimode coherent states. Regarding the growing interest in entangled states and their applications, these are possibilities for further research.

Acknowledgements

This work was supported by the Research Fund of Hungary (OTKA) under contract No. T034484.

References

[1] D. F. WALLS, Nature **306** (1983) 141.
[2] R. LOUDON and P. L. KNIGHT, J. Mod. Opt. **34** (1987) 709.
[3] U. LEONHARDT, *Measuring the quantum state of light*, Cambridge, 1997.
[4] S. M. BARNETT and P. M. RADMORE, *Methods in theoretical quantum optics,* Oxford, 1997.
[5] J. JANSZKY and A. V. VINOGRADOV, Phys. Rev. Lett. **64** (1990) 2771.
[6] P. ADAM, I. FÖLDESI, and J. JANSZKY, Phys. Rev. A **49** (1994) 1281.
[7] J. JANSZKY, P. DOMOKOS,and P. ADAM, Phys. Rev. A **48** (1993) 2213.
[8] S. SZABO, P. ADAM, J. JANSZKY, and P. DOMOKOS, Phys. Rev. A **53** (1996) 2698.
[9] Y. H. SHIH, A. V. SERGIENKO, M. H. RUBIN, T. E. KIESS, and C. O. ALLEY, Phys. Rev. A **50** (1994) 23.
[10] E. S. POLZIK, J. CARRI, and H. J. KIMBLE, Phys. Rev. Lett. **68** (1992) 3020.
[11] L.-A. WU, H. J. KIMBLE, J. L. HALL, and H. WU, Phys. Rev. Lett. **57** (1986) 2520.
[12] P. G. KWIAT, K. MATTLE, H. WEINFURTER, and A. ZEILINGER, Phys. Rev. Lett. **75** (1995) 4337.
[13] L. VAIDMAN, Phys. Rev. A **49** (1994) 1473.
[14] A. FURUSAWA, J. L. SØRENSEN, S. L. BRAUNSTEIN, C. A. FUCHS, H. J. KIMBLE, and E. S. POLZIK, Science **282** (1998) 706.
[15] S. L. BRAUNSTEIN and H. J. KIMBLE, Phys. Rev. Lett. **80** (1998) 869.
[16] G. J. MILBURN and S. L. BRAUNSTEIN, Phys. Rev. A **60** (1999) 937.
[17] T. OPATRNÝ, G. KURIZKI, and D.-G. WELSCH, Phys. Rev. A **61** (2000) 032302.
[18] S. J. VAN ENK, Phys. Rev. A **60** (1999) 5095.
[19] S. YU and C.-P. SUN, Phys. Rev. A **61** (2000) 022310.
[20] K. E. CAHILL, Phys. Rev. **138** (1965) B1566.
[21] A. WÜNSCHE, J. Phys. A: Math. Gen. **31** (1998) 8267.
[22] J. JANSZKY, M. KONIORCZYK, and A. GÁBRIS, Phys. Rev. A, accepted for publication.